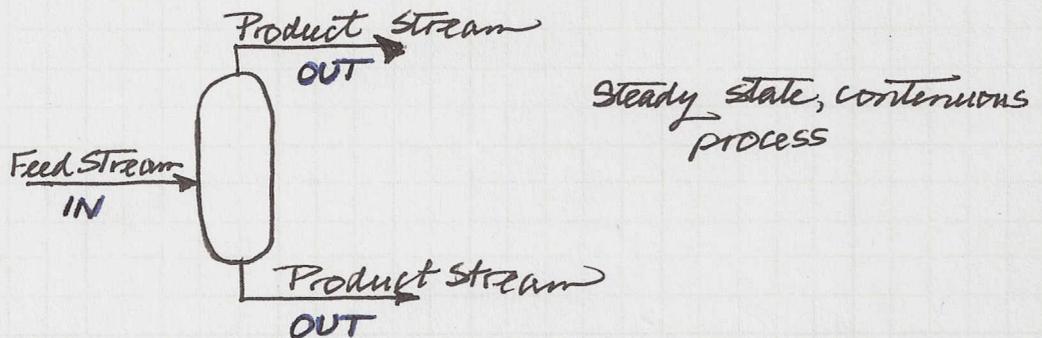


Chapter 4: § 4.1 - 4.3

Consider a single process vessel: a flash drum



Given inlet stream, we want to know what comes out, and how to get it. Material balances allow us to do this.

Start with some words:

- BATCH - all feed goes in at the beginning. all product leaves at the end.

Requires some more difficult equations.

- SEMI-BATCH - most industrial "batch" processes are semi-batch - the feed is added in several steps. The process still changes in time - it is TRANSIENT
- CONTINUOUS - feeds flow in and products flow out continuously. The process does not change with time if it is at STEADY STATE

So a STEADY STATE PROCESS does not change with time - a transient one does.

A TRANSIENT PROCESS changes with time

Now take the GENERAL MATERIAL BALANCE EQUATION

$$\underbrace{\text{ACCUMULATION}} = \underbrace{\text{IN} - \text{OUT}} + \underbrace{\text{GENERATION} - \text{CONSUMPTION}}$$

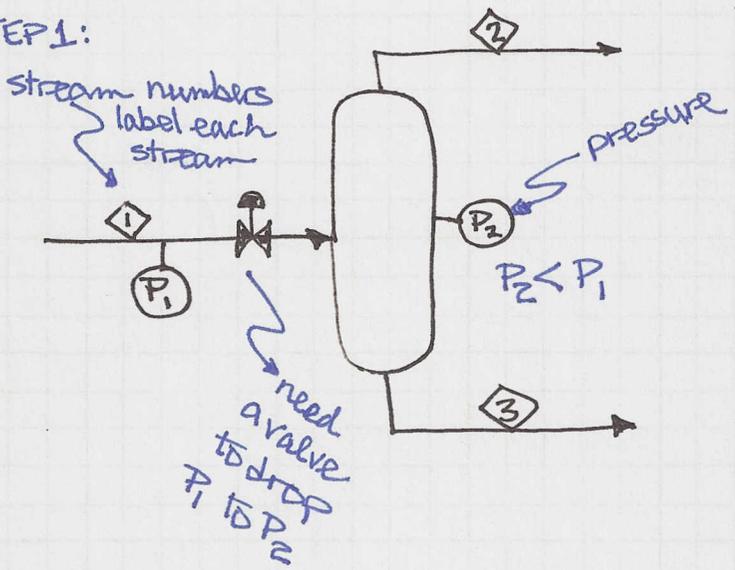
Changes with time = 0 if steady state

flows across the boundaries (= 0 if batch)

reaction terms = 0 if no reaction

Consider the flash drum: take a saturated liquid (eg pop) and drop the pressure suddenly (open the can). What happens? (CO₂ is released). This is how a flash drum works: drop the pressure to separate (eg CO₂) from (eg POP). In our plant example, we set up the next step of the material balance:

STEP 1:



STEP 2:

Steady state \Rightarrow accumulation = 0
 no reaction \Rightarrow generation = consumption = 0

∴ **IN = OUT**

stick with this for next two weeks.

NOTE that this starts to look a lot like a matrix!

STEP 3: MATERIAL BALANCE TABLE

COMPONENTS	STREAMS		
	1	2	3
iso butane (B)	x_{B1}	x_{B2}	x_{B3}
iso pentane (P)	x_{P1}	x_{P2}	x_{P3}
TOTAL	F_1	F_2	F_3
flow of B	$f_{B1} = x_{B1} F_1$	$f_{B2} = x_{B2} F_2$	$f_{B3} = x_{B3} F_3$
flow of P	$f_{P1} = x_{P1} F_1$	$f_{P2} = x_{P2} F_2$	$f_{P3} = x_{P3} F_3$

always required

need these for spreadsheet solutions

Applying the equation $IN = OUT$ gives

$F_1 = F_2 + F_3$ TOTAL OVERALL material balances

isobutane balance: $x_{B1} F_1 = x_{B2} F_2 + x_{B3} F_3$

isopentane balance: $x_{P1} F_1 = x_{P2} F_2 + x_{P3} F_3$

NOTE: must have mole fractions and molar flows OR mass fractions and mass flows to make the units work.

add the component balances:

$(x_{B1} + x_{P1}) F_1 = (x_{B2} + x_{P2}) F_2 + (x_{B3} + x_{P3}) F_3$

1.0 1.0 1.0

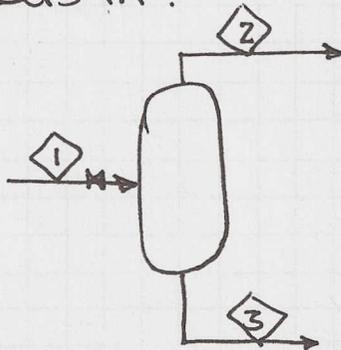
This is an important step - DO NOT SKIP!

so $F_1 = F_2 + F_3$ and there are 2 independent material balances

we say that the number of independent material balances is $n (1 \text{ TOTAL OVERALL} + \text{in mols} \text{ COMPONENT balances} - 1)$ where n is the number of components (in this case, $(1 + 2 - 1) = 2$) ✓.

Returning to the flash drum, now we put some numbers in.

STEP 1



STEP 2

- steady state.
 - no reaction
- $IN = OUT$

STEP 3

	MOLAR BASIS		
	1	2	3
B	0.55	0.91	x_{B3}
P	0.45	0.09	x_{P3}
flows (mol/hr)	F_1	F_2	F_3

85% of the isobutane fed is recovered in stream 2.

What is the composition of stream 3?

Solve for these given x_B

STEP 4 degrees of freedom analysis

List unknowns: $x_{B3}, x_{P3}, F_1, F_2, F_3 = 5$

Equations:

Degrees of freedom

unknowns ($F_1, F_2, F_3, x_{B3}, x_{P3}$)	5
MB	2
$\sum x_i$'s (3)	1
Process Spec	1
Basis	1
<hr/>	
d.f. = 5 - 5 = 0	

② 2 material balances
(pick from 1 overall and 2 components)

① 1 $\sum x_i = 1$

① 1 process spec: 85% of the B is recovered in stream 2.

4

$5 - 4 = 1$ degree of freedom
= 1 more unknown than equations available
So it is not possible to solve for all the unknowns in this problem.

No flow rates are given, so we can set a BASIS ← more on this later.

The BASIS is an arbitrary amount of material in one stream.
and solve the equations based on 100 moles of stream

1. Thus the equations become:

Overall MB: $F_1 = F_2 + F_3$ ~~Overall MB~~ ①

B-balance: $0.55 F_1 = 0.91 F_2 + x_{B3} F_3$ ~~B-balance~~ ②

$\sum x_i$: $x_{B3} + x_{P3} = 1.0$ ③

Process spec: $0.85 (0.55 F_1) = 0.91 F_2$ ④

Basis: $F_1 = 100 \text{ mol/hr}$ ⑤

Substitute ⑤ in ④ giving

$$F_2 = \frac{0.85 (0.55 (100 \text{ mol}))}{0.91} = 51.3 \text{ mol/hr}$$

Sub. $F_1 + F_2$ in ① giving $F_3 = (100 - 51.3) \text{ mol} = 48.7 \text{ mol/hr}$

Sub. in ② giving $x_{B3} = 0.17$

Sub in ③ giving $x_{P3} = 0.83$

hold this on the board until pg 7: BASIS

The BASIS: another look.

Suppose we set $F_1 = 100 \frac{\text{kmol}}{\text{hr}}$ instead of 100 mol.

What happens to the answer?

$$F_2 = 51.3 \frac{\text{kmol}}{\text{hr}}$$

$$F_3 = 48.7 \frac{\text{kmol}}{\text{hr}}$$

compositions unchanged.

Now try $F_1 = 500 \frac{\text{kmol}}{\text{hr}}$.

$$F_2 = 256.4 \frac{\text{kmol}}{\text{hr}}$$

$$F_3 = 243.6 \frac{\text{kmol}}{\text{hr}} = 5 \times (48.7 \frac{\text{kmol}}{\text{hr}})$$

compositions unchanged.

This is a very powerful result. The answer is scalable. We can solve the problem for an arbitrary basis (100 mols or kg of stream j). When we find out how big the plant will be, we only have to multiply the flowrates by the scaling factor = $\frac{\text{new flow}}{\text{basis}}$ = 5 in the example above, and the new MB is complete.

Check the solution using the P-balance:

$$0.45(100 \text{ mol}) = 0.09(51.3 \text{ mol}) + 0.83(48.7 \text{ mol})$$

$$45 \text{ mol} = 45.038 \text{ mol} \quad \text{OK.}$$

Final MB table

MOLAR BASIS

comp't	1	2	3
B	0.55	0.91	0.17
P	0.45	0.09	0.83
Flow (mol)	100	51.3	48.7

NOTE the separation into B-rich and P-rich streams

Summary of Material Balance

STEP 1: Draw a flowsheet

Label all streams with a stream number

STEP 2: Reduce the general balance equation, listing assumptions: eg SS, no rxn, $\therefore \text{IN} = \text{OUT}$

STEP 3: Construct the MB table with components, mole or mass fractions, and flows. Insert known values, solve by inspection when possible. The most general form of the table may also include components flows, $f_{ij} = x_{ij}F_j$. For this problem we would write:

Molar Basis
comp't

	1	2	3
B	0.55	0.91	x_{B3}
P	0.45	0.09	x_{P3}
Total flow (mol)	100	F_2	F_3
B-flow (mol)	55	$0.91F_2$	f_{B3}
P-flow (mol)	45	$0.09F_2$	f_{P3}

○ solved by inspection

} various forms of f_{ij} shown

STEP 4: Degrees of freedom analysis on the vessel.

- list unknowns
- list equations
 - n material balances (selected from n+1 available MB)
 - $\sum x_{ij} = 1.0$ for every stream, j, where 2 or more mol/mass fractions are unknown, you can write $\sum x_{ij} = 1.0$. If only one is unknown, solve directly and remove an unknown.
 - process specifications (e.g. recovery, conversion, % separation)
 - thermo or reaction data (later)
- ~~• basis~~
- if # unknowns = # equations, a solution can be obtained.

STEP 5: If # unknowns > # equations by 1, and no flowrates are specified, a BASIS for the calculation may be selected.

STEP 6: Solve the equations and check your answer using the last MB.